CP Violation in a Supersymmetric $SO(10) \times U(2)_F$ Model

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A model based on SUSY $SO(10)$ combined with $U(2)$ family symmetry constructed recently by the authors is generalized to include phases in the mass matrices leading to CP violation. In contrast with the commonly used effective operator approach, $\overline{126}$-dimensional Higgs fields are utilized to construct the Yukawa sector. R-parity symmetry is thus preserved at low energies. The *symmetric* mass textures arising from the left-right symmetry breaking chain of $SO(10)$ give rise to very good predictions for quark and lepton masses and mixings. The prediction for $\sin 2\beta$ agrees with the average of current bounds from BaBar and Belle. In the neutrino sector, our predictions are in good agreement with results from atmospheric neutrino experiments. Our model accommodates both the LOW and QVO solutions to the solar neutrino anomaly; the matrix element for neutrinoless double beta decay is highly suppressed. The leptonic analog of the Jarlskog invariant, J^l_{CP}, is predicted to be of $O(10^{-2})$.

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$SO(10)$ has long been thought to be an attractive candidate for a grand unified theory (GUT) for a number of reasons: First of all, it unifies all the 15 known fermions with the right-handed neutrino for each family into one 16-dimensional spinor representation. The seesaw mechanism then arises very naturally, and the non-zero neutrino masses can thus be explained. Since a complete quark-lepton symmetry is achieved, it has the promise for explaining the pattern of fermion masses and mixing. Because $B-L$ contained in $SO(10)$ is broken in symmetry breaking chain to SM, it also has the promise for baryogenesis. Recent atmospheric neutrino oscillation data from Super-Kamiokande indicates non-zero neutrino masses. This in turn gives very strong support to the viability of $SO(10)$ as a GUT group. Models based on $SO(10)$ combined with discrete or continuous family symmetry have been constructed to understand the flavor problem [1, 2]. Most of the models utilize "lopsided" mass textures which usually require more parameters and therefore are less constrained. Furthermore, the right-handed neutrino Majorana mass operators in most of these models are made out of $16_H \times 16_H$ which breaks the R-parity at a very high scale. We have recently constructed a realistic model based on supersymmetric $SO(10)$ combined with $U(2)$ family symmetry [1] (referred to as "CM" hereafter) which successfully predicts the low energy fermion masses and mixings. Since we utilize *symmetric* mass textures and $\overline{126}$-dimensional Higgs representations for the right-handed neutrino Majorana mass operator, our model is more constrained in addition to having R-parity conserved [3]. The aim of this paper is to generalize this model to include phases in the mass matrices which lead to CP violation. We first summarize our model followed by analytic analyses of the complex mass textures. And then the implications of our model for neutrino mixing and CP violation are presented.

The Model: The details of our model based on $SO(10) \times U(2)_F$ are contained in CM. The following is an outline of its salient features. In order to specify the superpotential uniquely, we invoke $Z_2 \times Z_2 \times Z_2$ discrete symmetry. The matter fields are

$$\psi_a \sim (16,2)^{-++} \quad (a=1,2), \qquad \psi_3 \sim (16,1)^{+++}$$

where $a = 1, 2$ and the subscripts refer to family indices; the superscripts $+/-$ refer to $(Z_2)^3$ charges. The Higgs fields which break $SO(10)$ and give rise to mass matrices upon acquiring VEV's are

$$\begin{array}{llllll} (10,1): & T_1^{+++}, & T_2^{-+-}, & T_3^{--+}, & T_4^{---}, & T_5^{+--} \\ (\overline{126},1): & & \overline{C}^{---}, & \overline{C}_1^{+++}, & \overline{C}_2^{++-} & \end{array} \tag{1}$$

Higgs representations 10 and $\overline{126}$ give rise to Yukawa couplings to the matter fields which are symmetric under the interchange of family indices. The left-right symmetry breaking chain of $SO(10)$ is

$$\begin{array}{rcl} SO(10) & \longrightarrow & SU(4) \times SU(2)_L \times SU(2)_R \\ & \longrightarrow & SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & \longrightarrow & SU(3) \times SU(2)_L \times U(1)_Y \\ & \longrightarrow & SU(3) \times U(1)_{EM} \end{array} \tag{2}$$

The $U(2)$ family symmetry is broken in two steps [4] and the mass hierarchy is produced using the Froggatt-Nielsen mechanism [5]:

$$U(2) \xrightarrow{\epsilon M} U(1) \xrightarrow{\epsilon' M} nothing \tag{3}$$

and M is the UV-cutoff of the effective theory above which the family symmetry is exact, and ϵM and $\epsilon' M$ are the VEV's accompanying the flavon fields given by

$$\begin{array}{llll} (1,2): & \phi_{(1)}^{++-}, & \phi_{(2)}^{+-+}, & \Phi^{-+-} \\ (1,3): & S_{(1)}^{+--}, & S_{(2)}^{---}, & \Sigma^{++-} \end{array} \tag{4}$$

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The various aspects of VEV's of Higgs and flavon fields are given in CM.

The superpotential for our model is

$$W = W_{Dirac} + W_{\nu_{RR}} + W_{flavon} \tag{5}$$

$$\begin{aligned} W_{Dirac} &= \psi_3\psi_3 T_1 + \frac{1}{M}\psi_3\psi_a \left(T_2\phi_{(1)} + T_3\phi_{(2)}\right) \\ &+ \frac{1}{M}\psi_a\psi_b \left(T_4 + \overline{C}\right) S_{(2)} + \frac{1}{M}\psi_a\psi_b T_5 S_{(1)} \\ W_{\nu_{RR}} &= \psi_3\psi_3\overline{C}_1 + \frac{1}{M}\psi_3\psi_a\Phi\overline{C}_2 + \frac{1}{M}\psi_a\psi_b\Sigma\overline{C}_2 \end{aligned} \tag{6}$$

where W_{flavon} is the superpotential involving only flavon fields which give rise to their VEV's given in Eq. (34) of CM [6]. The mass matrices then can be read from the superpotential to be

$$\begin{aligned} M_{u,\nu_{LR}} &= \begin{pmatrix} 0 & 0 & \left\langle 10_2^+\right\rangle \epsilon' \\ 0 & \left\langle 10_4^+\right\rangle \epsilon & \left\langle 10_3^+\right\rangle \epsilon \\ \left\langle 10_2^+\right\rangle \epsilon' & \left\langle 10_3^+\right\rangle \epsilon & \left\langle 10_1^+\right\rangle \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & r_2\epsilon' \\ 0 & r_4\epsilon & \epsilon \\ r_2\epsilon' & \epsilon & 1 \end{pmatrix} M_U \end{aligned} \tag{7}$$

$$\begin{aligned} M_{d,e} &= \begin{pmatrix} 0 & \left\langle 10_5^-\right\rangle \epsilon' & 0 \\ \left\langle 10_5^-\right\rangle \epsilon' & (1,-3)\left\langle \overline{126}^-\right\rangle \epsilon & 0 \\ 0 & 0 & \left\langle 10_1^-\right\rangle \end{pmatrix} \\ &= \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & (1,-3)p\epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} M_D \end{aligned} \tag{8}$$

where $M_U \equiv \left\langle 10_1^+\right\rangle$, $M_D \equiv \left\langle 10_1^-\right\rangle$, $r_2 \equiv \left\langle 10_2^+\right\rangle / \left\langle 10_1^+\right\rangle$, $r_4 \equiv \left\langle 10_4^+\right\rangle / \left\langle 10_1^+\right\rangle$ and $p \equiv \left\langle \overline{126}^-\right\rangle / \left\langle 10_1^-\right\rangle$. The right-handed neutrino mass matrix is

$$\begin{aligned} M_{\nu_{RR}} &= \begin{pmatrix} 0 & 0 & \left\langle \overline{126}_2^{\prime 0}\right\rangle \delta_1 \\ 0 & \left\langle \overline{126}_2^{\prime 0}\right\rangle \delta_2 & \left\langle \overline{126}_2^{\prime 0}\right\rangle \delta_3 \\ \left\langle \overline{126}_2^{\prime 0}\right\rangle \delta_1 & \left\langle \overline{126}_2^{\prime 0}\right\rangle \delta_3 & \left\langle \overline{126}_1^{\prime 0}\right\rangle \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R \end{aligned} \tag{9}$$

with $M_R \equiv \left\langle \overline{126}_1^{\prime 0}\right\rangle$. (This is one of the five sets of symmetric texture combinations (labeled set (v)) proposed by Ramond, Roberts and Ross [7].) Here, the superscripts $+/-/0$ refer to the sign of the hypercharge. It is to be noted that there is a factor of -3 difference between the (22) elements of mass matrices M_d and M_e. This is due to the CG coefficients associated with $\overline{126}$; as a consequence, we obtain the phenomenologically viable Georgi-Jarlskog relations [8].

In CM, VEV's were taken to be real. In general, all VEV's are complex leading to *spontaneous* CP violation which is the subject matter of this paper.

Analytical Analysis of Mass Textures: In our convention, the Yukawa couplings $Y_i,\ (i = u, d, e, \nu_{LR})$ are defined in the way that the left-handed fields are on the right, and the charge conjugate fields are on the left. In the quark sector, they read

$$\mathcal{L}_{mass} = -Y_u\overline{U}_R Q_L H_u - Y_d\overline{D}_R Q_L H_d + h.c.$$

And the charged current interaction is given by

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}(W_\mu^+\overline{U}_L\gamma_\mu D_L) + h.c.$$

Here we have written the Lagrangian in the weak basis. The Yukawa matrices are diagonalized by the bi-unitary transformations

$$\begin{aligned} Y_u^{diag} &= V_{u_R} Y_u V_{u_L}^\dagger = diag(y_u, y_c, y_t) \\ Y_d^{diag} &= V_{d_R} Y_d V_{d_L}^\dagger = diag(y_d, y_s, y_b) \end{aligned} \tag{10}$$

where V_R and V_L are the right-handed and left-handed rotations respectively, and all the eigenvalues y_i's are real and non-negative. To extract the left-handed (right-handed) rotation matrices, we need to consider the diagonalization of the hermitian quantity $Y^\dagger Y$ $(YY^\dagger)$. The Cabbibo-Kobayashi-Maskawa (CKM) matrix is then given by

$$V_{CKM} = V_{u_L} V_{d_L}^\dagger = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{11}$$

The unitary matrix V_{CKM} has in general 6 phases. By phase redefinition of the quarks, one can remove 5 of them. The remaining one phase is one of the sources for CP violation. A parameterization independent measure for the CP violation is the Jarlskog invariant [9], defined as $J_{CP}^q \equiv Im\{V_{11}V_{12}^* V_{21}^* V_{22}\}$. And the three angles of the CKM unitarity triangle are defined as

$$\begin{aligned} &\alpha \equiv Arg(-\tfrac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}), \qquad \beta \equiv Arg(-\tfrac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}) \\ &\gamma \equiv Arg(-\tfrac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}) \end{aligned} \tag{12}$$

In the lepton sector, the charged lepton Yukawa matrix is diagonalized by

$$Y_e^{diag} = V_{e_R} Y_e V_{e_L}^\dagger = diag(y_e, y_\mu, y_\tau) \tag{13}$$

And the effective light left-handed Majorana neutrino mass matrix (obtained after using the seesaw mechanism) is diagonalized by an unitary matrix $V_{\nu_{LL}}$:

$$M_{\nu_{LL}}^{diag} = V_{\nu_{LL}} M_{\nu_{LL}} V_{\nu_{LL}}^T = diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \tag{14}$$

where the eigenvalues $y_{e,\mu,\tau}$ and m_{ν_1,ν_2,ν_3} are real and non-negative. The leptonic mixing matrix, the Maki-Nakagawa-Sakata (MNS) matrix, can be parameterized by

$$U_{MNS} \equiv V_{e_L} V^\dagger_{\nu_{LL}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_l} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l} & s_{23}c_{13}e^{i\delta_l} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_l} & c_{23}c_{13}e^{i\delta_l} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\frac{\alpha_{21}}{2}} & \\ & & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix} \tag{15}$$

Note that the Majorana condition,

$$C(\overline{\nu}_j)^T = \nu_j \tag{16}$$

where C is the charge conjugate operator, forbids the rephasing of the Majorana fields. Therefore, we can only remove 3 of the 6 phases present in the unitary matrix U_{MNS} by redefing the charged lepton fields. Note that U_{MNS} is the product of an unitary matrix, analogous to the CKM matrix which has one phase (the so-called universal phase), δ_l, and a diagonal phase matrix which contains two phases (the so-called Majorana phases), α_{21} and α_{31}. The leptonic analog of the Jarlskog invariant, which measures the CP violation due to the universal phase, is given by

$$J^l_{CP} \equiv Im\{U_{\mu 2}U_{e3}U^*_{\mu 3}U^*_{e2}\} \tag{17}$$

For the Majorana phases, the rephasing invariant CP violation measures are [10]

$$S_1 \equiv Im\{U_{e1}U^*_{e3}\}, \qquad S_2 \equiv Im\{U_{e2}U^*_{e3}\} \tag{18}$$

From S_1 and S_2, one can then determine the Majorana phases

$$\begin{aligned} \cos\alpha_{31} &= 1 - 2\tfrac{S_1^2}{|U_{e1}|^2|U_{e3}|^2} \\ \cos(\alpha_{31} - \alpha_{21}) &= 1 - 2\tfrac{S_2^2}{|U_{e2}|^2|U_{e3}|^2} \end{aligned} \tag{19}$$

The Lagrangian is invariant under the following weak-basis phase transformations

$$\begin{aligned} U_L \to U_L^{'} = K_L^{+} U_L, &\qquad D_L \to D_L^{'} = K_L^{+} D_L \\ U_R \to U_R^{'} = K_R^{u} U_R, &\qquad D_R \to D_R^{'} = K_R^{d} D_R \end{aligned} \tag{20}$$

$$Y_u \to Y_u^{'} = K_R^u Y_u K_L, \quad Y_d \to Y_d^{'} = K_R^d Y_u K_L \tag{21}$$

where K_L^+, K_R^u, and K_R^d are diagonal phase matrices. We are interested in complex symmetric textures resulting from the $SO(10)$ relations:

$$\begin{aligned} Y_u &= \begin{pmatrix} 0 & 0 & ae^{i\gamma_a} \\ 0 & be^{i\gamma_b} & ce^{i\gamma_c} \\ ae^{i\gamma_a} & ce^{i\gamma_c} & e^{i\gamma} \end{pmatrix} d \\ Y_d &= \begin{pmatrix} 0 & ee^{i\gamma_e} & 0 \\ ee^{i\gamma_e} & fe^{i\gamma_f} & 0 \\ 0 & 0 & e^{i\gamma_h} \end{pmatrix} h \\ Y_{\nu_{LR}} &= \begin{pmatrix} 0 & 0 & ae^{i\gamma_a} \\ 0 & be^{i\gamma_b} & ce^{i\gamma_c} \\ ae^{i\gamma_a} & ce^{i\gamma_c} & e^{i\gamma} \end{pmatrix} d \\ Y_e &= \begin{pmatrix} 0 & ee^{i\gamma_e} & 0 \\ ee^{i\gamma_e} & -3fe^{i\gamma_f} & 0 \\ 0 & 0 & e^{i\gamma_h} \end{pmatrix} h \end{aligned}$$

with $a \simeq b \ll c \ll 1$ and $e \ll f \ll 1$. The weak-basis phase transformations mentioned above enable us to reduce the number of phases to two, $\theta \equiv (\gamma_b - 2\gamma_c - \gamma)$ and $\xi \equiv (\gamma_e - \gamma_f + \gamma_c - \gamma_a)$. Then we have

$$\begin{aligned} Y_{u,\nu_{LR}} &= \begin{pmatrix} 0 & 0 & a \\ 0 & be^{i\theta} & c \\ a & c & 1 \end{pmatrix} d \\ Y_{d,e} &= \begin{pmatrix} 0 & ee^{-i\xi} & 0 \\ ee^{i\xi} & (1,-3)f & 0 \\ 0 & 0 & 1 \end{pmatrix} h \end{aligned} \tag{22}$$

We then diagonalize the mass matrices analytically. In the leading order, the mass eigenvalues are given by

$$\begin{aligned} m_u &\simeq \frac{a^2bd}{|-be^{i\theta}+c^2|}v_u \\ m_c &\simeq |(-be^{i\theta}+c^2)|dv_u \\ m_t &\simeq dv_u \\ m_d &\simeq \frac{e^2fh}{e^2+f^2}v_d \\ m_s &\simeq \frac{(2e^2f+f^3)h}{e^2+f^2}v_d \\ m_b &= hv_d \\ m_e &\simeq \frac{3e^2fh}{e^2+9f^2}v_d \\ m_\mu &\simeq \frac{6e^2fh+27f^3h}{e^2+9f^2}v_d \\ m_\tau &= hv_d \end{aligned} \tag{23}$$

The CKM matrix elements are given by

$$\begin{aligned} V_{ud} &\simeq (1-\frac{1}{2}\frac{e^2}{f^2}) - (\frac{e}{f})(\frac{a}{c})e^{-i\delta_q} \\ V_{us} &\simeq \frac{e}{f} + \frac{a}{c}e^{-i\delta_q} \\ V_{ub} &\simeq ae^{-i\delta_q} \\ V_{cd} &\simeq -\frac{a}{c}e^{i\delta_q} - \frac{e}{f} \\ V_{cs} &\simeq (1-\frac{1}{2}\frac{e^2}{f^2}) - (\frac{e}{f})(\frac{a}{c})e^{i\delta_q} \\ V_{cb} &\simeq c \\ V_{td} &\simeq \frac{e}{f}c \\ V_{ts} &\simeq -c \\ V_{td} &\simeq 1 - \frac{1}{2}c^2 \end{aligned} \tag{24}$$

The phase δ_q is given by

$$\begin{aligned}\delta_q &\simeq \tan^{-1}(-\frac{a}{c}\sin(\theta^{'}-\xi)) \\ &- \tan^{-1}(-\frac{e}{f}\frac{a}{c}\sin(\theta^{'}-\xi)) + (\theta^{'}-\xi) \quad (25)\end{aligned}$$

where

$$\theta^{'} \simeq \tan^{-1}(b\sin\theta - 2\frac{b^3}{a^2}\sin^2\theta - 2\frac{b}{a^2}\sin\theta) \quad (26)$$

The form of the CKM matrix in Eq. (24) is also the one favored in [11].

In the lepton sector, similar diagonalization is carried out, and the charged lepton diagonalization matrix is

$$V_e = V_d\ (b \rightarrow -3b) \quad (27)$$

We obtain bimaximal mixing in the neutrino sector and $\Delta m^2_{atm} \gg \Delta m^2_{\odot}$ by choosing [1]

$$M_{\nu_{LL}} \simeq \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1 \\ t & 1 & 1 \end{pmatrix} \frac{d^2 v_u^2}{M_R} \quad (28)$$

In the basis where the Yukawa matrices take the forms of Eq. (22), this implies that the right-handed neutrino mass matrix, Eq. (9), has the following elements:

$$\begin{aligned}\delta_1 &\simeq \frac{a^2}{2a-2ac+c^2t} \\ \delta_2 &\simeq \frac{b^2te^{2i\theta}}{2a-2ac+c^2t} \\ \delta_3 &\simeq \frac{a(c-be^{i\theta})+bcte^{i\theta}}{2a-2ac+c^2t} \quad (29)\end{aligned}$$

The three eigenvalues of $M_{\nu_{LL}}$ are

$$\begin{aligned}m_{\nu_1} &\simeq (\frac{t}{\sqrt{2}} - \frac{t^2}{8})\frac{d^2v_u^2}{M_R} \\ m_{\nu_2} &\simeq (\frac{t}{\sqrt{2}} + \frac{t^2}{8})\frac{d^2v_u^2}{M_R} \\ m_{\nu_3} &\simeq (2 + \frac{t^2}{4})\frac{d^2v_u^2}{M_R} \quad (30)\end{aligned}$$

Note that since V_{e_L} is approximately an identity matrix, $U_{MNS} \approx V^{\dagger}_{\nu_{LL}}$. Consequently, with the present experimental status, it is not possible to make unique predictions for the leptonic CP violating phases from the fitting of the absolute values of the MNS matrix elements, as was done in the quark sector. We therefore assume that the light neutrino mass matrix to be real. And the leptonic CP violation will solely due to the phase present in Y_e.

RGE Analysis: We use the following inputs at $M_Z = 91.187\ GeV$:

$$\begin{aligned}m_u &= 2.32\ MeV(2.33^{+0.42}_{-0.45}) \\ m_c &= 677\ MeV(677^{+56}_{-61}) \\ m_t &= 182\ GeV(181^{+}_{-}13) \\ m_e &= 0.485\ MeV(0.486847) \\ m_\mu &= 103\ MeV(102.75) \\ m_\tau &= 1.744\ GeV(1.7467) \\ |V_{us}| &= 0.222(0.219-0.224) \\ |V_{ub}| &= 0.0039(0.002-0.005) \\ |V_{cb}| &= 0.036(0.036-0.046)\end{aligned}$$

where the values extrapolated from experimental data are given inside the parentheses [12]. These values correspond to the following set of input parameters at the GUT scale, $M_{GUT} = 1.03 \times 10^{16}\ GeV$:

$$a = 0.00246, \quad b = 3.50 \times 10^{-3}, \quad c = 0.0320$$

$$d = 0.650, \quad \theta = 0.110$$

$$e = 4.03 \times 10^{-3}, \quad f = 0.0195$$

$$h = 0.0686, \quad \xi = -0.720 \quad (31)$$

$$g_1 = g_2 = g_3 = 0.746 \quad (32)$$

the one-loop renormalization group equations for the MSSM spectrum with three right-handed neutrinos [13] are solved numerically down to the effective right-handed neutrino mass scale, M_R. At M_R, the seesaw mechanism is implemented. We then solve the two-loop RGE's for the MSSM spectrum [14] down to the SUSY breaking scale, taken to be $m_t(m_t) = 176.4\ GeV$, and then the SM RGE's from $m_t(m_t)$ to the weak scale, M_Z. We assume that $\tan\beta \equiv \frac{v_u}{v_d} = 10$, with $v_u^2 + v_d^2 = (246/\sqrt{2}\ GeV)^2$. At the weak scale M_Z, the predictions for $\alpha_i \equiv \frac{g_i^2}{4\pi}$ are

$$\alpha_1 = 0.01663, \quad \alpha_2 = 0.03374, \quad \alpha_3 = 0.1242$$

These values compare very well with the values extrapolated to M_Z from the experimental data [12], $(\alpha_1, \alpha_2, \alpha_3) = (0.01696, 0.03371, 0.1214^{+}_{-}0.0031)$. The predictions at the weak scale M_Z for the charged fermion masses, the CKM matrix elements and the CP violation measures, are summarized in TABLE I (after taking into account the SUSY threshold correction [15], $\Delta_b = -0.15$) along with the experimental values extrapolated to M_Z [12].

The current result from the atmospheric neutrino experiment is [16]

$$\begin{aligned}\Delta m^2_{23} &= (1.3-8)\times 10^{-3} eV^2 (\text{best fit} : 3.3 \times 10^{-3}) \\ \sin^2 2\theta &= 1\end{aligned}$$

In the solar neutrino sector, the current best fit from the solar neutrino experiment data for each of the solution is given by [17]:

TABLE I: The predictions for the charged fermion masses, the CKM matrix elements and the CP violation measures.

	experimental results extrapolated to M_Z	predictions at M_z
$\frac{m_s}{m_d}$	$17 \sim 25$	25
m_s	$93.4^{+11.8}_{-13.0} MeV$	$85.66 MeV$
m_b	$3.00^{+}_{-}0.11 GeV$	$3.147 GeV$
$\lvert V_{ud}\rvert$	$0.9745 - 0.9757$	0.9751
$\lvert V_{cd}\rvert$	$0.218 - 0.224$	0.2218
$\lvert V_{cs}\rvert$	$0.9736 - 0.9750$	0.9744
$\lvert V_{td}\rvert$	$0.004 - 0.014$	0.005358
$\lvert V_{ts}\rvert$	$0.034 - 0.046$	0.03611
$\lvert V_{tb}\rvert$	$0.9989 - 0.9993$	0.9993
J^q_{CP}	$(2.71^{+}_{-}1.12) \times 10^{-5}$	1.748×10^{-5}
$\sin 2\alpha$	$-0.95 \ - \ 0.33$	-0.8913
$\sin 2\beta$	$0.59^{+}_{-}0.14^{+}_{-}0.05$ (BaBar)	0.7416
	$0.99^{+}_{-}0.14^{+}_{-}0.06$(Belle)	
γ	$34^0 - 82^0$	34.55^0 $(0.6030 rad)$

LMA : (large angle MSW with larger mass squared)

$$\Delta m^2 = 3.2 \times 10^{-5} eV^2$$
$$\tan^2\theta = 0.33, \quad \sin^2 2\theta = 0.75$$

LOW : (large angle MSW with smaller mass squared)

$$\Delta m^2 = 1 \times 10^{-7} eV^2$$
$$\tan^2\theta = 0.67, \quad \sin^2 2\theta = 0.96$$

SMA : (small angle MSW)

$$\Delta m^2 = 5.0 \times 10^{-6} eV^2$$
$$\tan^2\theta = 0.0006, \quad \sin^2 2\theta = 0.0024$$

QVO : (quasi-vacuum oscillation)

$$\Delta m^2 = 8.6 \times 10^{-10} eV^2$$
$$\tan^2\theta = 1.5, \quad \sin^2 2\theta = 0.96$$

At present, none of these solutions can be ruled out [18]. The current bound on $|U_{e\nu_3}|$ from CHOOZ experiment is [19]

$$|U_{e\nu_3}| \leq 0.16$$

Our model favors both the LOW and QVO solution. The LOW solution is obtained with the following set of parameters:

$$M_R = 2.012 \times 10^{13} GeV, \quad t = 0.088 \tag{33}$$

which give rise to, using Eq. (29),

$$\begin{aligned} \delta_1 &= 0.001247 \\ \delta_2 &= 2.221 \times 10^{-4} e^{i\,(0.2201)} \\ \delta_3 &= 0.01648 e^{-i\,(0.001711)} \end{aligned}$$

This gives rise to three light neutrino masses

$$\begin{aligned} m_{\nu_1} &= 0.001711\, eV \\ m_{\nu_2} &= 0.001762\, eV \\ m_{\nu_3} &= 0.05438\, eV \end{aligned}$$

and the squared-mass differences are

$$\begin{aligned} \Delta m^2_{atm} &= 2.954 \times 10^{-3}\ eV^2 \\ \Delta m^2_{\odot} &= 1.769 \times 10^{-7}\ eV^2 \end{aligned} \tag{34}$$

And the MNS matrix is given by

$$|U_{MNS}| = \begin{pmatrix} 0.6743 & 0.7346 & 0.07497 \\ 0.5427 & 0.4322 & 0.7202 \\ 0.5008 & 0.5230 & 0.6897 \end{pmatrix}$$

This translates into

$$\sin^2 2\theta_{atm} \equiv 4|U_{\mu\nu_3}|^2(1-|U_{\mu\nu_3}|^2) = 0.9986$$
$$\sin^2 2\theta_{\odot} \equiv 4|U_{e\nu_2}|^2(1-|U_{e\nu_2}|^2) = 0.9937$$

and the MNS matrix element $|U_{e\nu_3}|$ is predicted to be 0.07497, in very good agreement with the experimental result. The leptonic analog of the Jarlskog invariant is predicted to be

$$J^l_{CP} \equiv Im\{U_{11}U^*_{12}U^*_{21}U_{22}\} = -0.008147$$

The matrix element for the neutrinoless double beta ($\beta\beta_{0\nu}$) decay, $|<m>|$, is given in terms of the rephasing invariant quantities by

$$\begin{aligned} |<m>|^2 &= m_1^2|U_{e1}|^4 + m_2^2|U_{e2}|^4 + m_3^2|U_{e3}|^4 \\ &+ 2m_1m_2|U_{e1}|^2|U_{e2}|^2\cos\alpha_{21} \\ &+ 2m_1m_3|U_{e1}|^2|U_{e3}|^2\cos\alpha_{31} \\ &+ 2m_2m_3|U_{e2}|^2|U_{e3}|^2\cos(\alpha_{31}-\alpha_{21}) \end{aligned} \tag{35}$$

The current upper bound for $|<m>|$ from the experiment is 0.2 eV [20]. The two Majorana phases $(\alpha_{31}, \alpha_{21})$ are $(0.9708, -1.326)$; they give rise to a highly suppressed $|<m>| = 1.359 \times 10^{-3}\ eV$[21]. The three heavy neutrino masses are given by

$$\begin{aligned} M_1 &= 9.412 \times 10^7 GeV \\ M_2 &= 1.486 \times 10^9 GeV \\ M_3 &= 2.013 \times 10^{13} GeV \end{aligned}$$

The QVO solution is obtained with:

$$M_R = 1.217 \times 10^{14} GeV, \quad t = 0.0142 \tag{36}$$

which give rise to,

$$\begin{aligned} \delta_1 &= 0.001267 \\ \delta_2 &= 3.641 \times 10^{-5} e^{i\,(0.2201)} \\ \delta_3 &= 0.01502 e^{-i\,(0.01074)} \end{aligned}$$

The three light neutrino masses are predicted to be

$$\begin{aligned} m_{\nu_1} &= 2.856 \times 10^{-4}\ eV \\ m_{\nu_2} &= 2.870 \times 10^{-4}\ eV \\ m_{\nu_3} &= 5.587 \times 10^{-2}\ eV \end{aligned}$$

and the squared-mass differences are

$$\begin{aligned}\Delta m^2_{atm} &= 3.122 \times 10^{-3}\, eV^2 \\ \Delta m^2_{\odot} &= 7.584 \times 10^{-10}\, eV^2\end{aligned}$$

And the MNS matrix is given by

$$|U_{MNS}| = \begin{pmatrix} 0.6794 & 0.7318 & 0.05310 \\ 0.5302 & 0.4513 & 0.7178 \\ 0.5072 & 0.5107 & 0.6942 \end{pmatrix}$$

This translates into

$$\sin^2 2\theta_{atm} = 0.9991, \quad \sin^2 2\theta_{\odot} = 0.9950$$

and the MNS matrix element $|U_{e\nu_3}|$ is predicted to be 0.05310, in very good agreement with the experimental result. The leptonic analog of the Jarlskog invariant is predicted to be

$$J^l_{CP} = -0.008110$$

The two Majorana phases $(\alpha_{31}, \alpha_{21})$ are $(1.385, -0.4976)$ and the matrix element for $\beta\beta_{0\nu}$ is predicted to be $| < m > | = 3.07 \times 10^{-4}$ eV. The three heavy right-handed Majorana neutrino masses are

$$\begin{aligned}M_1 &= 3.699 \times 10^{7} GeV \\ M_2 &= 2.341 \times 10^{10} GeV \\ M_3 &= 1.218 \times 10^{14} GeV\end{aligned}$$

Our model can also produce the LMA solution by properly choosing the values for M_R and t; however, the prediction for $U_{e\nu_3}$ violates the experimental bound, leading to the elimination of the LMA solution in our model.

A few words concerning baryonic asymmetry are in order. Even though the sphaleron effects destroy baryonic asymmetry, it could be produced as an asymmetry in the generation of $(B - L)$ at a high scale because of lepton number violation due to the decay of heavy right-handed Majorana neutrinos [22], which in turn is converted into baryonic asymmetry due to sphalerons. But in our model this mechanism produces baryonic asymmetry of $O(10^{-13})$ which is too small to account for the observed value of $(1.7 - 8.3) \times 10^{-11}$ [23], reasons being that the mass of the lightest right-handed Majorana neutrino is too small *and* the $1-3$ family mixing of right-handed neutrinos is too large, leading, in essence, to the violation of the out-of-equilibrium condition required by Sakharov [24]. So a mechanism other than leptogenesis is required to explain baryonic asymmetry.

Summary: We have generalized our recently constructed model based on SUSY $SO(10)$ combined with $U(2)$ family symmetry to include phases in the mass matrices leading to CP violation. In contrast with the commonly used effective operator approach, $\overline{126}$-dimensional Higgs fields are utilized to construct the Yukawa sector. R-parity symmetry is thus preserved at low energies. The *symmetric* mass textures arising from the left-right symmetry breaking chain of $SO(10)$ give rise to very good predictions for quark and lepton masses and mixings. The prediction for $\sin 2\beta$ agrees with the average of current bounds from BaBar and Belle. In the neutrino sector, our predictions are in good agreement with results from atmospheric neutrino experiments. Our model allows both the LOW and QVO solutions to the solar neutrino anomaly, and the matrix element for $\beta\beta_{0\nu}$ decay is highly suppressed. The leptonic analog of the Jarlskog invariant, J^l_{CP}, is predicted to be of $O(10^{-2})$. It is interesting to note that, in the Yukawa sector, the model predicts $(12+3)$ masses, $(6+3)$ mixing angles and $(4+3)$ phases (- the additional 3's in the parentheses refer to the right-handed neutrino sector) in terms of nine parameters given in Eq. (31) and t and M_R, a total of eleven parameters.

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